Closing Wed: HW_1A, 1B, 1C
Read newsletter and postings.
Read sections 5.2, 5.3, and 5.4.

Entry Task: Approximate the area under $f(x) = 1 + x^2$ from $x = 2$ to $x = 3$ using Riemann sums with $n = 4$ and right endpoints.

Step 1: $\Delta x = \frac{b-a}{n} =$

Step 2: $x_0 = a =$

$x_1 = a + \Delta x =$

$x_2 = a + 2\Delta x =$

$x_3 = a + 3\Delta x =$

$x_4 = a + 4\Delta x =$

Step 3: Plug in right endpoints to get rectangle heights

Area ≈

$f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x$

Pattern?

$x_i = a + i\,\Delta x =$

$$\sum_{i=1}^{4} f(x_i)\Delta x =$$

General pattern in terms of *n*?

Another Example:

Using sigma notation, write down the general Riemann sum definition of the area from x = 5 to x = 7 under

$$f(x) = 3x + \sqrt{x}$$

$$\Delta x = \frac{b - a}{n} =$$

$$x_i = a + i\,\Delta x$$

$$\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x =$$

Velocity/Distance & Reimann Sums

When velocity is a *constant*:

Distance = Velocity · Time

If velocity is not constant, we can break the problem and approximate by assuming that velocity is constant over each subdivision.

Example:

You are accelerating in a car. You get the following measurements:

t (sec)	0	0.5	1.0	1.5	2.0
v(t) (ft/s)	0	6.2	10.8	14.9	18.1

Estimate the distance traveled by the car traveled from 0 to 2 seconds.

5.2 The Definite Integral

Def'n:
We define the **definite integral of f(x) from x = a to x = b** by

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x\,,$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$.

Some Basic Definite Integral Rules:

1. $\int_a^b c\,dx = (b-a)c$

2. $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

3. $\int_a^b cf(x)\,dx = c\int_a^b f(x)dx$

   and

   $\int_a^b f(x) + g(x)\,dx$

   $= \int_a^b f(x)dx + \int_a^b g(x)\,dx$

4. $\int_b^a f(x)dx = -\int_a^b f(x)dx$

Examples:

1. $\int_4^{10} 5\,dx =$

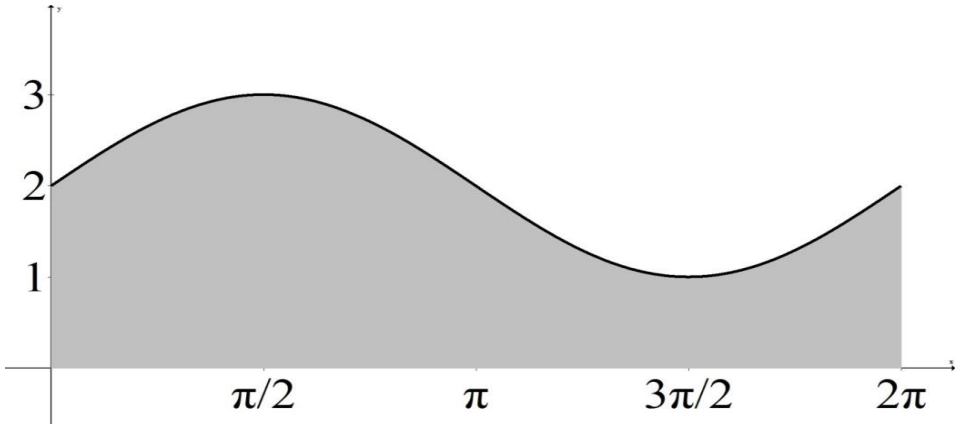
2. $\int_0^3 x^2dx + \int_3^7 x^2dx =$

3. $\int_0^4 5x + 3\,dx =$

4. $\int_3^1 x^3dx = -\int_1^3 x^3\,dx$

Note on quick bounds (HW_1C: 9,10)

$$m(b-a) \leq \int_a^b \mathrm{f(x)}dx \leq M(b-a)$$



Example: Consider the area under

f(x) = sin(x) + 2

on the interval *x = 0* to *x = 2π.*

(a) What is the max of f(x)? (label M)
(b) What is the min of f(x)? (label m)

(c) Draw **one** rectangle that contains all the shaded area? What can you conclude?
(d) Draw **one** rectangle that is completely inside the shaded area? Conclusion?